Delta-Hedged Gains and the Risk-Neutral Moments

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JEL classification: G13

Keywords: Delta-hedged gains; Skewness; Kurtosis

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Abstract

We investigate the well-documented underperformance of delta-hedged option portfolios in relation to ex ante moments of the stock market's return distribution. Using a sample of S&P 500 index options, we find that delta-hedged option gains decrease with ex ante volatility, in support of negative volatility risk premium. Moreover, the delta-hedged gains are negatively associated with skewness and kurtosis among call options, but positively associated with the higher moments among put options. These results suggest that investors pay premium for call options in anticipation of a positive jump, while they pay premium for put options in anticipation of a negative jump.

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1. Introduction

It is well-documented that risk-neutral distribution of the stock market returns is left-skewed and fat-tailed (See, for example, Rubinstein (1994), Jackwerth (2000), and Bakshi, Kapadia, and Madan (2003)). In the tradition of Jackwerth and Rubinstein (1996) and Bates (2000), the skewness and kurtosis of the risk-neutral distribution are linked to the characteristics of market jumps such that the fear of market crashes imparts negative skewness and positive kurtosis to the distribution. Many researchers have examined the importance of these higher moments, as well as volatility, in pricing securities.[1] In particular, with respect to option pricing, models incorporating stochastic volatility and jump of equity returns have been proposed.[2]

A related but different strand of literature focuses on estimating the risk-neutral moments and examining their information contents. In particular, Bakshi and Madan (2000), Carr and Madan (2001), and Bakshi, Kapadia, and Madan (2003) develop the method to extract the moments from the cross section of market option prices, not relying on any specific option pricing model. The moments inferred from the current option prices are forward-looking, reflecting the most recent market evaluation for future return distribution.

This paper investigates the role of ex ante risk-neutral moments in explaining the delta-hedged option gains. The delta-hedged option gains represent the excess dollar returns of an option after hedging out the option exposure to the underlying asset movement. The negativity of the delta-hedged gains is well-documented and often explained by the negative volatility

[1] Building on Kraus and Litzenberger (1976), an impressive range of researchers, including Harvey and Siddique (1999, 2000), Dittmar (2002), and Chabi-Yo (2008), have investigated three- and four-factor capital asset pricing models.
[2] Examples are Duffie, Pan and Singleton (2000), Bakshi, Cao and Chen (1997, 2000), Bakshi and Cao (2003), Bates (1991, 2000), Heston (1993), Hull and White (1987), Johnson and Shanno (1987), Kim and Kim (2004, 2005), Scott (1987), Stein and Stein (1991) and Wiggins (1987).

risk premium (See Coval and Shumway (2001), Pan (2002), Bakshi and Kapadia (2003), and Eraker, Johannes, and Polson (2003)). Investors are willing to pay a premium for buying market volatility because it provides hedges against a market decline coinciding with a volatility rise. Apart from the volatility, however, the possibility of jump events affects option prices (Jackwerth and Rubinstein (1996) and Bates (2000)). If option prices incorporate a nonzero jump risk premium, then the ex ante skewness and kurtosis, which reflect the investors' evaluation of possible future stock market crashes, may influence the losses on the delta-hedged option strategy.

The idea that the higher-order moments might play an important role in explaining the delta-hedged option gains is not original to this study. Bakshi and Kapadia (2003) explore the effect of the risk-neutral skewness and kurtosis on the delta-hedged gains, but their analysis is restricted to the near-the-money call options for the period from 1988 to 1995. After the period examined in Bakshi and Kapadia (2003), the stock market has gone through several downward spikes including the Asian financial crisis in 1997, the burst of Internet bubble in 2000, and the recent financial crisis in 2008. If investors' attitudes toward the risk have changed over the period under consideration, risk premium incorporated in option prices might have altered. Thus, we revisit the relation between the delta-hedged gains and the risk-neutral moments by examining both call and put options for a wide range of moneyness from 1996 through 2014.

For our empirical analysis, we define the delta-hedged option gain as the change in the value of a portfolio that buys one Standard and Poor's (S&P) 500 index option and involves positions in the S&P 500 index so that the portfolio is not sensitive to index movement. The portfolio is rebalanced daily over the maturity of the option. We also compute daily moments of market returns implied in S&P 500 index option prices following the model-free methodology of Bakshi, Kapadia, and Madan (2003). Consistent with previous work on the negative market volatility risk premium, we find that delta-hedged option gains are significantly negative and decrease with ex ante volatility.

Our principal finding is that the delta-hedged gains are negatively associated with ex ante skewness and kurtosis among call options, but positively associated with the higher moments among put options. The same pattern holds for all moneyness and maturity categories. We interpret these results to imply that investors would willing to pay a premium to hold call options in their portfolio when they anticipate positive jumps because call prices react positively to positive jump shocks. Similarly, when anticipating negative jumps, investors would willing to pay a premium to hold put options.

Our finding on the relation between the delta-hedged gain and higher moments is the opposite of the conclusion by Bakshi and Kapadia (2003) that the impact of skewness and kurtosis is less clear. They find that skewness is positively associated with the delta-hedged gains among the near-the-money calls, but only marginally significant. What differentiates our results from their work is the different sample periods, implying that risk premium incorporated in option prices have changed going through several market crashes.

The empirical results of this article strengthen the view that investors in the equity index option market consider higher moments in market returns. Specifically, our finding of different relation between higher moments and delta-hedged gains across call and put options complements the previous study focusing on a representative call option. Our results indicate that impacts of higher moments on options vary with exposures to jump risk, consistent with the view that writing index call options hedges positive jump risk while purchasing put options hedges negative jump risk.

The rest of the paper is organized as follows. In the next section, we discuss the data used in our analysis and detail the method to estimate ex ante measures of volatility, skewness, and kurtosis, proposed in Bakshi, Kapadia, and Madan (2003). In Section 3, we examine how the gains from delta-hedged option portfolios are related to ex ante moments of the stock market's risk-neutral probability distributions. Section 4 concludes our study by summarizing the results.

2. Data and Ex Ante Risk-Neutral Moments

2.1. Data

Our sample period covers January 1996 to December 2014. The S&P 500 index option data come from the Ivy OptionMetrics database. The data include the end-of-day bid and ask quotes, implied volatilities, open interest, and trading volume for the S&P 500 index options. We also obtain data on the daily S&P 500 index values and estimates of dividend yields, as well as the term structures of zero-coupon interest rates constructed from LIBOR quotes and Eurodollar futures prices.

The following rules are applied in order to filter the data. To avoid microstructure-related bias, we exclude options whose prices are less than $3/8, as well as options with maturities less than two weeks or longer than 60 days. We eliminate all options that have implied volatilities less than 1% or more than 100% in order to minimize the impact of recording errors. We also discard all option observations that violate no-arbitrage bounds. Finally, we define the moneyness of the option as $y \equiv Se^{(r-\delta)\tau}/K - 1$, where S is the underlying index value, K is the strike price, τ is the time-to-maturity, r is the risk-free rate, and δ is the dividend yield. Then, we omit deep away-from-the-money options by restricting y to the $\pm$ 10% range. Our option sampling procedure results in 327,670 calls and 336,226 puts.

We divide options into two maturity groups and eight moneyness categories, following Bakshi and Kapadia (2003): options with maturity 14-30 days and 31-60 days and with ranges of moneyness from -10% to +10% by 2.5%. Table 1 reports the number of days when the options data are observed during our sample period, the average daily number of options, price (i.e., midpoint of the bid and ask quotes), bid-ask spread, trading volume, and open interest, separately for options grouped over maturity and moneyness combinations. Panel A is for call options and Panel B is for put options.

[Table 1 about here]

Panel A of Table 1 shows that on average the number of call options traded on each day is about 46, ranging from three to eight depending on the moneyness category and having the largest value in the category of $y \in [-2.5\%, 0\%)$. Trading volume and open interest show that options with $y \in [-2.5\%, -2.5\%]$ has the largest value, indicating that near-the-money options are most actively traded. Not surprisingly, option prices increase monotonically with the moneyness. For example, deep out-of-the-money (OTM) call options with $y \in [-10\%, -7.5\%)$ have the average price of 1.93, while deep in-the-money (ITM) call options with $y \in [7.5\%, 10\%)$ have the average price of 104.49. Bid-ask spreads in dollar amount also increase with the moneyness. In term of relative values to option prices, however, the percentage spreads decrease with moneyness. We find similar patterns in Panel B for put options. Put option prices and dollar bid-ask spreads decrease monotonically as y increases. Also, near-the-money or slightly OTM put options with $y \in [0\%, 5\%]$ are most actively traded.

2.2.Ex Ante Risk-Neutral Moments

Bakshi and Madan (2000) show that the continuum of options can span any payoff function with bounded expectation. Based on this insight, Bakshi, Kapadia, and Madan (2003) formalize a mechanism to extract model-free estimates of risk-neutral moments from a set of option prices with different strike prices. Specifically, the date *t* risk-neutral moments of the index return over the period $[t, t+\tau]$ can be calculated as

$$VAR(t,\tau) = e^{r\tau}V(t,\tau) - \mu(t,\tau)^2 \tag{1}$$

$$SKEW(t,\tau) = \frac{e^{r\tau}W(t,\tau) - 3\mu(t,\tau)e^{r\tau}V(t,\tau) + 2\mu(t,\tau)^3}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^{3/2}} \tag{2}$$

$$KURT(t,\tau) = \frac{e^{r\tau}X(t,\tau) - 4\mu(t,\tau)W(t,\tau) + 6e^{r\tau}\mu(t,\tau)^2V(t,\tau) - \mu(t,\tau)^4}{[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2]^2} \tag{3}$$

where

$$\mu(t,\tau) = e^{r\tau} - 1 - e^{r\tau}V(t,\tau)/2 - e^{r\tau}W(t,\tau)/6 - e^{r\tau}W(t,\tau)/24 \tag{4}$$

and $V(t,\tau)$, $W(t,\tau)$, and $X(t,\tau)$ are the linear combinations of OTM call option prices $C(t,\tau,K)$ and put option prices $P(t,\tau,K)$, with time-to-maturity τ and strike price K:

$$V(t,\tau) = \int_{S_t}^{\infty} \frac{2\big(1 - ln(K/S_t)\big)}{K^2} C(t,\tau,K)dK + \int_0^{S_t} \frac{2\big(1 - ln(K/S_t)\big)}{K^2} P(t,\tau,K)dK, \tag{5}$$

$$\begin{aligned} W(t,\tau) = \int_{S_t}^{\infty} \frac{6ln(K/S_t) - 3\big(ln(K/S_t)\big)^2}{K^2} C(t,\tau,K)dK \\ + \int_0^{S_t} \frac{6ln(K/S_t) - 3\big(ln(K/S_t)\big)^2}{K^2} P(t,\tau,K)dK, \end{aligned} \tag{6}$$

$$X(t,\tau) = \int_{S_t}^{\infty} \frac{12\big(ln(K/S_t)\big)^2 - 4\big(ln(K/S_t)\big)^3}{K^2} C(t,\tau,K)dK$$
$$+ \int_{0}^{S_t} \frac{12\big(ln(K/S_t)\big)^2 - 4\big(ln(K/S_t)\big)^3}{K^2} P(t,\tau,K)dK. \quad (7)$$

It is important to note that the Bakshi, Kapadia, and Madan (2003)'s measures have the advantage of being forward-looking in contrast with traditional historical moment estimates. Since they are inferred from the contemporaneous option prices data, they impart investors' expectations of future index values. Also, the Bakshi, Kapadia, and Madan (2003)'s option-implied moments are model-free and therefore, we do not need to specify any particular option pricing model.

A challenge for estimating the model-free option-implied moments using expressions (5) to (7) is that options are available only over a finite range of strike prices while an infinite continuum of strike prices is needed. To address this issue, we follow Jiang and Tian (2005) and Chang, Christoffersen, and Jacobs (2013). Specifically, we apply the curve-fitting method to implied volatilities and use the endpoint implied volatilities to extrapolate for options with strike prices beyond the available range. We estimate the daily moments only for days that have at least two OTM calls and two OTM puts available for a given maturity. We use a trapezoidal approximation to estimate the integral in expressions (5) to (7) using discrete data. We calculate the implied moments over a 30-day horizon using linear interpolation between maturities.

Figure 1 shows the daily estimates of volatility, skewness, and kurtosis for a fixed 30-day horizon. All three time series vary significantly through time. The volatility peaks in 2008, during the financial crisis, then declines through 2014. The skewness is always negative and the kurtosis is always larger than three.[3] The skewness stays relatively flat through 2008 after which it tends to decline, while the kurtosis increases during the same period, more than doubling from 2008 through 2014. We can find the same patterns in Table 2, which reports the 5th, 50th, and 95th percentiles of daily estimates of option implied moments over time for each year during the sample period. In sum, the results in Figure 1 and Table 2 clearly show that index returns are strongly negatively skewed and fat-tailed, consistent with previous studies[4]. Most notably, these departures from normality become more pronounced after the 2008 financial crisis.

[Figure 1 about here]

[Table 2 about here]

3. Empirical Results

3.1. Statistical Properties of Delta-Hedged Gains

We begin by computing delta-hedged option gain, which is the change in the value of a self-financing portfolio consisting of a long position in the option, hedged by the underlying index so that the portfolio value is not sensitive to index value movement. Our calculation of the discrete delta-hedged gains follows Bakshi and Kapadia (2003). Specifically, the hedge is

[3] One exception is that kurtosis estimate on July 13, 2009 is less than three. On that day, the estimates of volatility, skewness, and kurtosis are 25.29%, -1.027, and 1.923, respectively.
[4] See, for example, Bakshi, Cao and Chen (1997, 2000), Bakshi, Kapadia and Madan (2003), Dennis and Mayhew (2002), Derman (1999) and Rubinstein (1994).

rebalanced at each of the dates until expiration.[5] The daily rebalanced delta-hedged call option gain over the period $[t, t+\tau]$, $\pi_{t,t+\tau}$, is computed as

$$\pi_{t,t+\tau} = C_{t+\tau} - C_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} r_n \left(C_{t_n} - \Delta_{t_n} S_{t_n}\right) \frac{\tau}{N}, \tag{8}$$

where Δ_{t_n} is the delta of the option and r_n is annualized risk-free rate at each of the dates $t_n, n = 0,1, \dots, N-1$ (where we define $t_0 = t, t_N = t+\tau$). Similarly, daily rebalanced delta-hedged put option gain is computed as

$$\pi_{t,t+\tau} = P_{t+\tau} - P_t - \sum_{n=0}^{N-1} \Delta_{t_n} \left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} r_n \left(P_{t_n} - \Delta_{t_n} S_{t_n}\right) \frac{\tau}{N}. \tag{9}$$

Table 3 presents for call options the averages of (i) the dollar delta-hedged gains ($\pi_{t,t+\tau}$), (ii) the delta-hedged gains scaled by the index level ($\pi_{t,t+\tau}/S_t$), and (iii) the delta-hedged gains scaled by the option price ($\pi_{t,t+\tau}/C_t$), for each maturity and moneyness category. The averages of delta-hedged gains are negative and statistically different from zero for every moneyness and maturity groups, indicating that delta-hedged option strategy loses money regardless of moneyness and maturity.

[Table 3 about here]

Looking down a column of Table 3, we can see that the dollar delta-hedged call gains become more negative as the moneyness increases. For example, the results of call options with maturity 14-30 days in Panel A show that the delta-hedged loss from deep OTM calls

[5] Because the exercise style of the S&P 500 index options is European, our results are not affected by the complication that arises owing to the early exercise feature of American options.

with $y \in [-10\%, -7.5\%)$ is \$0.22, while that from deep ITM calls with $y \in [7.5\%, 10\%)$ is \$6.02. When scaled by the index level, the delta-hedged loss increases monotonically from 1% of the index level to 50% as we move from OTM calls to ITM calls. However, after adjusting the different price levels of options across moneyess by scaling the dollar gains by the price, we can see that the magnitude of the delta-hedged loss relative to the option price decreases with moneyness. In Panel A, for example, the delta-hedged loss of deep OTM calls corresponds to about 60% of the option price, while that of deep ITM calls corresponds to about 6% of the option price. As reported in the next column labelled '(Ask-Bid)/C', the percentage bid-ask spread decreases from 50% for deep OTM calls to 2% for deep ITM calls. When compared to the percentage bid-ask spread, the delta-hedged loss relative to the option price appears large in all moneyness and maturity groups. The rightmost column labelled '$1_{\pi<0}$' shows the average frequency of negative delta-hedged gains among each group of options, indicating that more than 60% of delta-neutral option strategies lose money.

A comparison of the results in Panels A and B of Table 3 shows that delta-hedged gains become more negative with maturity. For example, in the results for options with maturity 31-60 days shown in Panel B, the delta-hedged loss for options with $y \in [-2.5\%, 0\%)$ is \$3.70, larger than that of \$1.99 for options with maturity 14-30 days presented in Panel A. This is consistent with the theoretical prediction that the sensitivity of option prices with respect to the volatility (a so-called 'vega') increases with maturity, resulting in more negative risk premium for longer-maturity options.

Table 4 reports the delta-hedged gains for put options. We can see that delta-hedged put gains are significantly negative for every moneyness and maturity groups. The dollar delta-hedged put gains become less negative as y increases. For example, the results of puts with maturity 14-30 days in Panel A show that the delta-hedged loss from deep ITM puts with $y \in [-10\%, -7.5\%)$ is \$9.59 (corresponding to 80% of the index level), while that from deep OTM puts with $y \in [7.5\%, 10\%)$ is \$1.59 (corresponding to 14% of the index level). After scaling the dollar gains by the option price, however, the delta-hedged loss relative to the option price increases with y. For example, in Panel A, the delta-hedged loss of deep ITM puts corresponds to about 8% of the option price, while that of deep OTM puts corresponds to about 46% of the put price. The results for options maturing in 31-60 days in Panel B show similar patterns, but delta-hedged losses generally become larger.

[Table 4 about here]

When compared to the results in Table 3, delta-hedged loss is more severe for put options than for call options. The delta-hedged loss relative to the option prices is larger for puts than for calls, allowing for the average percentage bid-ask spread. Also, the frequency of negative delta-hedged gains tends to be higher for puts than for calls. About 66% to 86% of delta-hedged put strategies lose money depending on the maturity and moneyness, while the frequency of the negative gains from calls ranges from 60% to 68%.

In sum, the results from both call and put options with a wide range of moneyness show that the delta-hedged strategy of index options underperforms zero after the period examined in Bakshi and Kapadia (2003).

3.2.Delta-Hedged Gains and the Volatility Risk Premium

The evidence on the underperformance of delta-hedged option strategy is often explained by the negative volatility risk premium (Coval and Shumway (2001), Pan (2002), Bakshi and Kapadia (2003), and Eraker, Johannes, and Polson (2003)). Investors pay a premium for buying market volatility because it provides hedges against a market decline coinciding with a volatility rise. To the extent that the delta-hedged loss is driven by volatility risk premium, the expected volatility level should affect the delta-hedged loss. In this section, we examine the relation between ex ante volatility and the delta-hedged loss.

In Table 5, we estimate for call options the following time-series regression of the delta-hedged gains on the lagged gains and ex ante volatility:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \varepsilon_t, \tag{10}$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t is the model-free estimate of risk-neutral volatility, constructed following Bakshi, Kapadia, and Madan (2003). We have included a lagged gains in the regression to mitigate the problem of serial correlation in the residuals following Bakshi and Kapadia (2003). We also compute the t-statistics by Generalized Method of Moments (GMM) using the approach of Newey and West (1987) to account for time-series dependence.

Unlike Bakshi and Kapadia (2003), where historical measures of volatility are used, we employ ex ante measure of volatility extracted from the current option prices. The ex ante measure has the advantage of being forward-looking and imparting concurrent investors' expectations for future market returns in contrast with the historical estimates.

The results in Table 5 show that coefficient estimates on volatility are negative for call options with all maturity and moneyness categories. The magnitude of coefficient estimates shows decreasing patterns over moneyness. For the options maturing within 14-30 days in

Panel A, the coefficient estimates on volatility decrease from -0.160 for $y \in [-7.5\%, -5\%)$ to -0.016 for $y \in [5\%, 7.5\%)$. Also, the negative coefficients lose the statistical significance for near-the-money and OTM calls. In Panel B, the results for the options maturing within 31-60 days show the similar patterns, but the magnitude and statistical significance of negative coefficients on volatility become stronger in general. Coefficient estimates on the lagged gains are close to one throughout both panels, indicating strong serial correlation of delta-hedged gains.

[Table 5 about here]

Table 6 reports the estimation results of regression of Eq. (10) for put options. We can see that coefficient estimates on volatility are negative for puts with most maturity and moneyness categories. As put options move toward being OTM, the negative relation between the delta-hedged gains and volatility tend to lose its significance. The significance of negative coefficients on volatility becomes stronger with maturity. In comparison to the results in Table 5, the magnitude and statistical significance of negative coefficients on volatility is generally larger for put options.

[Table 6 about here]

Overall, our evidence of negative relation between delta-hedged gains and ex ante volatility corroborates the negative volatility risk premium to the extent that higher volatility implies a more negative volatility risk premium.

3.3. Delta-Hedged Gains and Jump Exposures

Researchers have recognized that the possibility of jump events, as well as volatility, affects option prices (See for example Jackwerth and Rubinstein (1996) and Bates (2000)). If option prices incorporate a nonzero jump risk premium, then the ex ante skewness and kurtosis, which reflect the investors' evaluation of possible future stock market crashes, may influence the losses on the delta-hedged option strategy. In this section, we examine how ex ante skewness and kurtosis translate into the performance of the delta-hedged option strategy.

In Table 7, we estimate for call options the following time-series regression of the delta-hedged gains on the lagged gains and ex ante higher moments:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \beta_3 SKEW_t + \beta_4 KURT_t + \varepsilon_t, \tag{11}$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t, $SKEW_t$, and $KURT_t$ are the model-free estimates of the risk-neutral volatility, skewness, and kurtosis, respectively, constructed following Bakshi, Kapadia, and Madan (2003).

[Table 7 about here]

The results in Table 7 show that coefficient estimates on skewness and kurtosis are negative for call options with all maturity and moneyness categories. In Panel A of the 14-30 days option results, the coefficient estimates on skewness range from -0.02 to -0.94, with the tendency of ITM calls to have more negative estimates. The statistical significance is weak in OTM calls and deep ITM calls. However, in Panel B of the 31-60 days option results, the statistical significance of negative impacts of higher moments on the delta-hedged gains become larger in comparison with those in Panel A. For example, t-statistics of coefficient estimates on skewness range from -2.92 to -5.24 in Panel B, while those in Panel A range from -1.27 to -4.25. The coefficients on volatility are still negative after controlling for higher moments throughout. Overall, the results show that skewness and kurtosis are important sources of the loss of the delta-hedged call strategy apart from volatility.

Table 8 reports the estimation results of regression of Eq. (11) for put options. We can see that coefficient estimates on volatility are mostly negative for puts after taking into account higher moments. Most importantly, coefficient estimates on skewness and kurtosis are positive in all maturity and moneyness groups. Although the magnitude and statistical significance of the positive relation vary with the characteristics of options, all suggest that higher values of skewness and kurtosis translate into lesser underperformance of the delta-hedged put strategy. This is in stark contrast to the results for call options in Table 7 that delta-hedged call gains are negatively related to the higher moments. What causes the results for call and put options to differ is their different exposures to the market jump. Buyers of index call options gain when a positive jump occurs, while buyers of put options gain from negative jumps. Thus, investors pay premium for call options when they expect a positive jump, while they pay premium for put options when they expect a negative jump.

[Table 8 about here]

Taken together, our results show that delta-hedged gains are negatively associated with skewness and kurtosis among call options, but positively associated with the higher moments among put options. This is the key new finding of our paper, implying that investors pay premium for options depending on the direction of jumps they anticipate.

3.4. Subsample Analysis

In this section, we repeat the previous analyses for two sub-periods: the first spans 1996 to 2008, and the second covers 2009 to 2014. These subsamples are chosen because the financial crisis of 2008 incurred dramatic changes in the real economy and many researchers often classify periods as before- and after-2008. Also, as we discussed from Figure 1 in Section 2.2, the option-implied moments indicate that the distribution of index returns becomes more negatively skewed and fat-tailed after the 2008 financial crisis.

In Table 9, we repeat the analysis in Table 7 for two sub-periods. Specifically, we run the time-series regression of the delta-hedged call gains on the lagged gains and ex ante moments for each sub-period. The table shows that in the first sub-period, skewness and kurtosis, as well as volatility, affect the delta-hedged gains negatively, consistent with the results shown in Table 7. However, in the second sub-period, the negative impacts of the skewness and kurtosis disappear. This holds irrespective of the maturity of options examined.

The results of put options, presented in Table 10, show that the relation between the delta-hedged gains and ex ante moments does not change before and after 2008. In both sub-periods, volatility is negatively related to the delta-hedged gains and higher moments are positively associated with the delta-hedged gains.

[Table 9 about here]

[Table 10 about here]

In sum, the results in Tables 9 and 10 imply that investors have paid premium for calls in anticipation of a positive jump before the 2008 financial crisis, but after that, this tendency disappears. In contrast, investors are still willing to pay premium for puts in anticipation of a negative jump after the financial crisis.

4. Conclusion

This paper provides a comprehensive study of delta-hedged gains from stock index options in relation to the ex ante risk-neutral moments of market returns. Given several market crashes in the last two decades, it is worth revisiting the question of whether and how ex ante moments in market returns influence the delta-hedge option gains. Using S&P 500 index option data, we find that the delta-hedged option strategy loses money in general and its underperformance is more pronounced with higher ex ante volatility, confirming that index option prices incorporate the negative volatility risk premium. The key new finding is that the delta-hedged gains are negatively associated with skewness and kurtosis among call options, but positively associated with the higher moments among put options. This holds for a wide range of options' moneyness and maturity. These results suggest that investors pay premium for call options in anticipation of a positive jump, while they pay premium for put options in anticipation of a negative jump.

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Table 1. Summary statistics

The data on S&P 500 index options are from the OptionMetrics Ivy database over the sample period from January 1996 to December 2014. The table reports the number of days when the options data are observed during our sample period, the average daily number of options, price (i.e., midpoint of the bid and ask quotes), bid-ask spread, trading volume, and open interest, separately for options with maturity 14-30 days and 31-60 days and with eight different ranges of moneyness. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$, where S is the underlying index value, K is the strike price, τ is the time-to-maturity, r is the risk-free rate, and δ is the dividend yield. Panel A is for call options and Panel B is for put options.

Panel A: Call options

Time-to-maturity	Moneyness y	Number of days	Daily Number of options	Price	Bid-ask spread	Trading volume	Open interest
14-30 days	-10% to -7.5%	761	3.0	1.93	0.58	4,522	48,032
	-7.5% to -5%	1,894	4.1	2.37	0.55	5,464	49,810
	-5% to -2.5%	2,579	7.1	5.11	0.72	10,367	82,335
	-2.5% to 0%	2,646	7.9	13.96	1.16	14,730	98,199
	0% to 2.5%	2,615	7.3	31.61	1.78	7,416	82,792
	2.5% to 5%	2,547	6.5	54.98	2.14	1,173	48,282
	5% to 7.5%	2,367	5.4	79.86	2.29	529	25,990
	7.5% to 10%	1,979	4.4	104.49	2.47	248	15,371
31-60 days	-10% to -7.5%	3,379	4.1	3.05	0.73	2,818	33,875
	-7.5% to -5%	4,391	6.0	5.32	0.85	4,924	49,568
	-5% to -2.5%	4,558	7.5	11.27	1.21	8,027	68,812
	-2.5% to 0%	4,558	7.6	22.87	1.65	10,400	72,023
	0% to 2.5%	4,557	7.3	39.93	1.99	7,418	66,924
	2.5% to 5%	4,547	6.7	60.78	2.18	1,206	44,771
	5% to 7.5%	4,509	5.9	83.51	2.29	460	27,484
	7.5% to 10%	4,390	5.0	106.51	2.41	237	14,829

Panel B: Put options

Time-to-maturity	Moneyness y	Number of days	Daily Number of options	Price	Bid-ask spread	Trading volume	Open interest
14-30 days	-10% to -7.5%	1,072	4.5	122.91	3.16	226	17,422
	-7.5% to -5%	1,543	5.8	85.41	2.85	619	22,292
	-5% to -2.5%	2,291	7.0	53.47	2.34	1,224	30,819
	-2.5% to 0%	2,623	7.8	30.20	1.85	6,797	54,938
	0% to 2.5%	2,651	7.6	16.25	1.25	16,183	100,910
	2.5% to 5%	2,649	7.1	9.30	0.98	12,299	105,263
	5% to 7.5%	2,599	6.6	5.73	0.79	9,928	94,644
	7.5% to 10%	2,518	6.0	3.82	0.67	9,160	84,532
31-60 days	-10% to -7.5%	2,172	5.6	117.57	2.94	215	16,278
	-7.5% to -5%	3,181	6.4	86.55	2.70	356	19,410
	-5% to -2.5%	4,300	7.1	58.04	2.29	1,071	27,917
	-2.5% to 0%	4,558	7.6	38.37	2.01	6,468	48,560
	0% to 2.5%	4,557	7.3	25.34	1.71	13,647	81,586
	2.5% to 5%	4,549	6.7	17.20	1.49	8,976	82,275
	5% to 7.5%	4,522	6.1	12.07	1.27	7,197	72,661
	7.5% to 10%	4,495	5.7	8.74	1.10	5,778	60,172

Table 2. Descriptive statistics of option-implied moments of S&P 500 index returns

This table reports the model-free estimates of risk-neutral moments implied in S&P 500 index option prices. We compute the risk-neutral moments following the procedure in Bakshi, Kapadia, and Madan (2003), detailed in Section 2.2. The reported values are the 5th percentile, median, and 95th percentile of daily estimates for volatility, skewness, and kurtosis by year, over the sample period from January 1996 through December 2014.

	Volatility			Skewness			Kurtosis		
Year	P5	P50	P95	P5	P50	P95	P5	P50	P95
1996	0.13	0.16	0.20	-1.95	-1.66	-1.20	5.72	7.71	9.55
1997	0.19	0.21	0.33	-2.28	-1.23	-0.60	4.16	5.94	11.22
1998	0.18	0.24	0.42	-2.57	-2.11	-1.65	7.00	10.77	14.29
1999	0.20	0.25	0.30	-2.13	-1.74	-1.37	6.61	8.51	12.29
2000	0.18	0.23	0.29	-1.67	-1.28	-0.79	4.98	6.32	8.49
2001	0.20	0.24	0.36	-2.19	-1.38	-1.04	5.45	7.19	12.40
2002	0.19	0.27	0.41	-1.98	-1.38	-0.98	4.91	6.83	10.28
2003	0.17	0.20	0.34	-2.04	-1.21	-0.81	4.60	6.30	11.72
2004	0.13	0.15	0.19	-2.11	-1.63	-1.02	5.93	8.56	13.28
2005	0.11	0.13	0.16	-2.61	-1.86	-1.33	7.20	10.41	16.38
2006	0.11	0.12	0.18	-2.80	-1.96	-1.39	7.27	10.49	18.33
2007	0.10	0.17	0.27	-2.68	-2.05	-1.23	5.02	10.04	15.97
2008	0.19	0.25	0.73	-2.01	-1.33	-0.98	4.48	6.11	10.61
2009	0.22	0.30	0.50	-2.51	-1.81	-1.28	6.75	9.93	15.73
2010	0.17	0.22	0.35	-2.87	-2.08	-1.61	7.95	11.28	19.25
2011	0.16	0.22	0.42	-3.31	-2.33	-1.71	8.27	12.78	24.96
2012	0.15	0.18	0.23	-3.45	-2.19	-1.55	7.52	12.66	30.12
2013	0.13	0.14	0.18	-3.47	-2.18	-1.64	8.14	12.13	27.68
2014	0.12	0.14	0.21	-4.49	-3.16	-2.30	12.93	22.80	43.44
All	0.12	0.20	0.38	-3.06	-1.79	-1.01	5.12	9.09	22.01

Table 3. Delta-hedged gains for S&P 500 index call options

This table reports the average delta-hedged gains for S&P 500 index call options, separately for options with maturity 14-30 days and 31-60 days and with eight different ranges of moneyness. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Following Bakshi and Kapadia (2003), we compute the delta-hedged gain as the change in the value of a self-financing portfolio consisting of a long call position, hedged by a short position in the underlying index. The hedge is rebalanced at each of the dates until expiration. The daily rebalanced delta-hedged call option gain over the period $[t, t+\tau]$, $\pi_{t,t+\tau}$, is computed as

$$\pi_{t,t+\tau} = C_{t+\tau} - C_t - \sum_{n=0}^{N-1} \Delta_{t_n}\left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} r_n\left(C_{t_n} - \Delta_{t_n} S_{t_n}\right)\frac{\tau}{N},$$

where Δ_{t_n} is the delta of the option and r_n is annualized risk-free rate at each of the dates $t_n, n = 0,1, \dots, N-1$ (where we define $t_0 = t, t_N = t+\tau$). The table presents the averages of (i) the dollar delta-hedged gains ($\pi_{t,t+\tau}$), (ii) the delta-hedged gains scaled by the index level ($\pi_{t,t+\tau}/S_t$), and (iii) the delta-hedged gains scaled by the option price ($\pi_{t,t+\tau}/C_t$). The corresponding standard errors are reported in parentheses. For comparison, we report the option percentage spread as the difference between ask and bid quotes divided by the midpoint. $1_{\pi<0}$ is the proportion of delta-hedged gains with $\pi_{t,t+\tau} < 0$.

Moneyness y	π (in $)	π/S (in %)	π/C (in %)	(Ask-Bid)/C (in %)	$1_{\pi<0}$ (%)
Panel A: 14-30 days					
-10% to -7.5%	-0.22	-0.01	-58.99	50.54	66.89
	(0.13)	(0.01)	(4.54)		
-7.5% to -5%	-0.61	-0.05	-61.71	39.64	66.26
	(0.09)	(0.01)	(2.92)		
-5% to -2.5%	-1.17	-0.10	-37.57	21.09	63.75
	(0.11)	(0.01)	(2.02)		
-2.5% to 0%	-1.99	-0.18	-15.65	9.11	61.72
	(0.16)	(0.01)	(1.11)		
0% to 2.5%	-3.19	-0.27	-10.15	5.79	64.02
	(0.23)	(0.02)	(0.69)		
2.5% to 5%	-3.92	-0.33	-7.28	3.99	63.53
	(0.31)	(0.03)	(0.58)		
5% to 7.5%	-4.54	-0.37	-5.69	2.93	60.29
	(0.40)	(0.03)	(0.54)		
7.5% to 10%	-6.02	-0.50	-5.87	2.38	62.05
	(0.51)	(0.04)	(0.54)		
Panel B: 31-60 days					
-10% to -7.5%	-1.18	-0.12	-56.60	47.97	67.53
	(0.08)	(0.01)	(2.02)		
-7.5% to -5%	-1.76	-0.17	-45.02	30.23	66.59
	(0.09)	(0.01)	(1.57)		
-5% to -2.5%	-2.86	-0.27	-30.27	14.25	66.92
	(0.13)	(0.01)	(1.10)		
-2.5% to 0%	-3.70	-0.33	-17.22	7.73	66.76
	(0.16)	(0.01)	(0.69)		
0% to 2.5%	-4.57	-0.39	-11.62	5.13	68.31
	(0.19)	(0.02)	(0.47)		
2.5% to 5%	-5.20	-0.44	-8.48	3.65	67.14
	(0.23)	(0.02)	(0.38)		
5% to 7.5%	-6.19	-0.51	-7.28	2.77	65.51
	(0.28)	(0.02)	(0.35)		
7.5% to 10%	-8.55	-0.71	-8.25	2.27	64.01
	(0.34)	(0.03)	(0.35)		

Table 4. Delta-hedged gains for S&P 500 index put options

This table reports the average delta-hedged gains for S&P 500 index put options, separately for options with maturity 14-30 days and 31-60 days and with eight different ranges of moneyness. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Following Bakshi and Kapadia (2003), we compute the delta-hedged gain as the change in the value of a self-financing portfolio consisting of a long put position, hedged by a long position in the underlying index. The hedge is rebalanced at each of the dates until expiration. The daily rebalanced delta-hedged put option gain over the period $[t, t+\tau]$, $\pi_{t,t+\tau}$, is computed as

$$\pi_{t,t+\tau} = P_{t+\tau} - P_t - \sum_{n=0}^{N-1} \Delta_{t_n}\left(S_{t_{n+1}} - S_{t_n}\right) - \sum_{n=0}^{N-1} r_n\left(P_{t_n} - \Delta_{t_n} S_{t_n}\right)\frac{\tau}{N},$$

where Δ_{t_n} is the delta of the option and r_n is annualized risk-free rate at each of the dates $t_n, n = 0,1,\ldots,N-1$ (where we define $t_0 = t, t_N = t+\tau$). The table presents the averages of (i) the dollar delta-hedged gains ($\pi_{t,t+\tau}$), (ii) the delta-hedged gains scaled by the index level ($\pi_{t,t+\tau}/S_t$), and (iii) the delta-hedged gains scaled by the option price ($\pi_{t,t+\tau}/P_t$). The corresponding standard errors are reported in parentheses. For comparison, we report the option percentage spread as the difference between ask and bid quotes divided by the midpoint. $1_{\pi<0}$ is the proportion of delta-hedged gains with $\pi_{t,t+\tau} < 0$.

Moneyness y	π (in $)	π/S (in %)	π/P (in %)	(Ask-Bid)/P (in %)	$1_{\pi<0}$ (%)
Panel A: 14-30 days					
-10% to -7.5%	-9.59	-0.80	-8.11	2.58	65.95
	(0.79)	(0.06)	(0.81)		
-7.5% to -5%	-8.01	-0.67	-9.26	3.33	67.47
	(0.62)	(0.05)	(0.87)		
-5% to -2.5%	-6.45	-0.52	-11.82	4.45	71.63
	(0.41)	(0.03)	(0.87)		
-2.5% to 0%	-4.44	-0.35	-14.74	6.24	73.85
	(0.28)	(0.02)	(0.95)		
0% to 2.5%	-3.25	-0.27	-20.15	7.86	75.90
	(0.19)	(0.02)	(1.07)		
2.5% to 5%	-2.68	-0.23	-30.01	11.58	80.86
	(0.14)	(0.01)	(1.16)		
5% to 7.5%	-2.10	-0.18	-39.32	16.49	83.46
	(0.13)	(0.01)	(1.30)		
7.5% to 10%	-1.59	-0.14	-46.00	22.48	86.26
	(0.12)	(0.01)	(1.39)		
Panel B: 31-60 days					
-10% to -7.5%	-13.48	-1.13	-11.22	2.51	72.19
	(0.66)	(0.05)	(0.73)		
-7.5% to -5%	-11.74	-0.97	-13.27	3.11	75.29
	(0.51)	(0.04)	(0.75)		
-5% to -2.5%	-9.72	-0.82	-16.50	4.00	77.77
	(0.36)	(0.03)	(0.75)		
-2.5% to 0%	-7.59	-0.64	-20.00	5.40	80.10
	(0.27)	(0.02)	(0.80)		
0% to 2.5%	-5.67	-0.48	-23.02	7.03	80.21
	(0.22)	(0.02)	(0.83)		
2.5% to 5%	-4.50	-0.39	-27.89	9.24	83.05
	(0.17)	(0.01)	(0.88)		
5% to 7.5%	-3.68	-0.32	-34.08	11.91	84.45
	(0.13)	(0.01)	(0.85)		
7.5% to 10%	-2.75	-0.24	-38.66	15.02	86.18
	(0.11)	(0.01)	(0.90)		

Table 5. Delta-hedged gains and volatility risk premium: S&P 500 index call options

This table reports the estimation results for call options from the regression of delta-hedged gains on the lagged delta-hedged gains and the risk-neutral volatility:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t is the model-free estimate of risk-neutral volatility, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The regressions are performed separately for each maturity and moneyness category. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with time-to-maturity between 14 and 30 days and Panel B is for options with time-to-maturity between 31 and 60 days.

Moneyness y	Intercept	Lagged Gains	VOL	R^2 (%)
Panel A: 14-30 days				
-10% to -7.5%	0.105	0.915	-0.451	85.4
	(3.06)	(16.78)	(-3.58)	
-7.5% to -5%	0.027	0.928	-0.160	90.5
	(2.47)	(32.16)	(-2.78)	
-5% to -2.5%	0.015	0.953	-0.090	93.9
	(1.18)	(92.74)	(-1.29)	
-2.5% to 0%	0.021	0.961	-0.084	94.7
	(1.59)	(127.31)	(-1.16)	
0% to 2.5%	0.018	0.978	-0.052	96.6
	(1.50)	(234.42)	(-0.82)	
2.5% to 5%	0.026	0.985	-0.047	97.6
	(1.89)	(368.09)	(-0.65)	
5% to 7.5%	0.040	0.978	-0.016	97.5
	(2.33)	(274.38)	(-0.19)	
7.5% to 10%	0.057	0.977	0.072	96.1
	(2.80)	(184.24)	(0.78)	
Panel B: 31-60 days				
-10% to -7.5%	0.110	0.931	-0.552	93.8
	(5.95)	(60.98)	(-6.70)	
-7.5% to -5%	0.063	0.958	-0.347	96.0
	(5.84)	(103.91)	(-6.15)	
-5% to -2.5%	0.050	0.966	-0.271	96.4
	(5.21)	(166.18)	(-5.33)	
-2.5% to 0%	0.038	0.967	-0.204	96.3
	(4.75)	(175.44)	(-4.77)	
0% to 2.5%	0.032	0.970	-0.176	96.1
	(3.69)	(209.50)	(-3.80)	
2.5% to 5%	0.017	0.977	-0.095	96.6
	(1.84)	(250.16)	(-2.07)	
5% to 7.5%	0.017	0.977	-0.090	96.7
	(1.63)	(279.74)	(-1.77)	
7.5% to 10%	0.011	0.972	-0.048	96.2
	(0.61)	(240.29)	(-0.51)	

Table 6. Delta-hedged gains and volatility risk premium: S&P 500 index put options

This table reports the estimation results for put options from the regression of delta-hedged gains on the lagged delta-hedged gains and the risk-neutral volatility:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t is the model-free estimate of risk-neutral volatility, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The regressions are performed separately for each maturity and moneyness category. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with time-to-maturity between 14 and 30 days and Panel B is for options with time-to-maturity between 31 and 60 days.

Moneyness y	Intercept	Lagged Gains	VOL	R^2 (%)
Panel A: 14-30 days				
-10% to -7.5%	0.105	0.921	-0.613	89.8
	(2.79)	(69.96)	(-3.12)	
-7.5% to -5%	0.184	0.945	-0.616	93.5
	(6.65)	(127.69)	(-5.61)	
-5% to -2.5%	0.168	0.939	-0.525	93.5
	(6.61)	(92.77)	(-5.05)	
-2.5% to 0%	0.052	0.981	-0.154	97.0
	(3.36)	(208.75)	(-2.04)	
0% to 2.5%	0.022	0.974	-0.061	96.5
	(2.25)	(171.91)	(-1.19)	
2.5% to 5%	0.017	0.970	-0.040	96.4
	(1.98)	(119.20)	(-0.89)	
5% to 7.5%	0.004	0.975	0.031	97.4
	(0.47)	(127.68)	(0.76)	
7.5% to 10%	-0.017	0.977	0.161	97.5
	(-1.56)	(66.06)	(2.73)	
Panel B: 31-60 days				
-10% to -7.5%	0.105	0.964	-0.530	95.1
	(2.95)	(183.06)	(-3.46)	
-7.5% to -5%	0.150	0.970	-0.510	95.6
	(5.34)	(229.62)	(-4.74)	
-5% to -2.5%	0.152	0.964	-0.606	95.8
	(6.89)	(239.61)	(-6.40)	
-2.5% to 0%	0.070	0.969	-0.377	96.6
	(3.96)	(227.98)	(-4.12)	
0% to 2.5%	0.046	0.972	-0.230	97.0
	(5.05)	(235.93)	(-5.13)	
2.5% to 5%	0.035	0.966	-0.174	96.8
	(4.81)	(157.89)	(-4.42)	
5% to 7.5%	0.011	0.956	-0.062	95.9
	(1.36)	(128.36)	(-1.44)	
7.5% to 10%	-0.002	0.946	0.029	95.3
	(-0.16)	(79.67)	(0.52)	

Table 7. Delta-hedged gains and jump exposures: S&P 500 index call options

This table reports the estimation results for call options from the regression of delta-hedged gains on the lagged delta-hedged gains, the risk-neutral volatility, skewness, and kurtosis:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \beta_3 SKEW_t + \beta_4 KURT_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t, $SKEW_t$, and $KURT_t$ are the model-free estimates of the risk-neutral volatility, skewness, and kurtosis, respectively, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The regressions are performed separately for each maturity and moneyness category. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with time-to-maturity between 14 and 30 days and Panel B is for options with time-to-maturity between 31 and 60 days.

Moneyness y	Intercept	Lagged Gains	VOL	SKEW	KURT	R^2 (%)
Panel A: 14-30 days						
-10% to -7.5%	0.074	0.918	-0.469	-0.044	-0.004	85.5
	(2.51)	(17.05)	(-3.48)	(-1.36)	(-1.17)	
-7.5% to -5%	0.007	0.928	-0.159	-0.020	-0.002	90.5
	(0.42)	(31.91)	(-2.63)	(-1.27)	(-1.26)	
-5% to -2.5%	-0.015	0.953	-0.084	-0.030	-0.003	93.9
	(-0.65)	(90.36)	(-1.13)	(-1.77)	(-1.73)	
-2.5% to 0%	-0.026	0.960	-0.082	-0.052	-0.005	94.7
	(-1.01)	(119.94)	(-1.07)	(-2.37)	(-2.44)	
0% to 2.5%	-0.047	0.978	-0.059	-0.079	-0.008	96.6
	(-2.09)	(227.61)	(-0.86)	(-3.71)	(-3.71)	
2.5% to 5%	-0.041	0.986	-0.064	-0.090	-0.009	97.6
	(-1.56)	(369.50)	(-0.80)	(-4.25)	(-4.41)	
5% to 7.5%	-0.025	0.978	-0.042	-0.093	-0.010	97.5
	(-0.70)	(274.31)	(-0.44)	(-3.37)	(-3.94)	
7.5% to 10%	0.002	0.977	0.029	-0.094	-0.010	96.1
	(0.03)	(187.86)	(0.29)	(-1.45)	(-1.85)	
Panel B: 31-60 days						
-10% to -7.5%	0.093	0.931	-0.571	-0.035	-0.004	93.8
	(4.66)	(63.15)	(-6.73)	(-2.92)	(-3.18)	
-7.5% to -5%	0.035	0.958	-0.352	-0.037	-0.004	96.0
	(2.64)	(106.85)	(-6.13)	(-3.79)	(-3.87)	
-5% to -2.5%	0.011	0.965	-0.272	-0.045	-0.004	96.4
	(0.86)	(159.80)	(-5.22)	(-4.19)	(-4.14)	
-2.5% to 0%	-0.012	0.967	-0.206	-0.059	-0.006	96.3
	(-0.74)	(165.70)	(-4.55)	(-4.14)	(-3.98)	
0% to 2.5%	-0.025	0.970	-0.184	-0.076	-0.008	96.1
	(-1.43)	(194.04)	(-3.67)	(-4.58)	(-4.46)	
2.5% to 5%	-0.058	0.977	-0.105	-0.097	-0.010	96.6
	(-2.97)	(232.95)	(-2.07)	(-5.24)	(-5.03)	
5% to 7.5%	-0.063	0.977	-0.107	-0.109	-0.011	96.7
	(-2.76)	(273.33)	(-1.92)	(-5.14)	(-5.18)	
7.5% to 10%	-0.062	0.972	-0.080	-0.114	-0.013	96.3
	(-1.78)	(236.27)	(-0.81)	(-4.05)	(-4.44)	

Table 8. Delta-hedged gains and jump exposures: S&P 500 index put options

This table reports the estimation results for put options from the regression of delta-hedged gains on the lagged delta-hedged gains, the risk-neutral volatility, skewness, and kurtosis:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \beta_3 SKEW_t + \beta_4 KURT_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t, $SKEW_t$, and $KURT_t$ are the model-free estimates of the risk-neutral volatility, skewness, and kurtosis, respectively, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The regressions are performed separately for each maturity and moneyness category. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with time-to-maturity between 14 and 30 days and Panel B is for options with time-to-maturity between 31 and 60 days.

Moneyness y	Intercept	Lagged Gains	VOL	SKEW	KURT	R^2 (%)
Panel A: 14-30 days						
-10% to -7.5%	0.242	0.921	-0.569	0.197	0.021	89.8
	(2.00)	(69.02)	(-2.69)	(1.96)	(2.01)	
-7.5% to -5%	0.291	0.945	-0.583	0.148	0.016	93.5
	(4.34)	(129.54)	(-5.02)	(2.37)	(2.52)	
-5% to -2.5%	0.182	0.939	-0.499	0.035	0.004	93.5
	(3.21)	(92.11)	(-4.54)	(0.70)	(0.94)	
-2.5% to 0%	0.107	0.982	-0.158	0.059	0.005	97.0
	(3.50)	(206.02)	(-1.92)	(2.80)	(2.83)	
0% to 2.5%	0.065	0.973	-0.055	0.055	0.006	96.5
	(3.85)	(167.42)	(-1.07)	(4.41)	(4.64)	
2.5% to 5%	0.054	0.968	-0.045	0.041	0.004	96.4
	(3.98)	(120.03)	(-0.94)	(4.44)	(4.36)	
5% to 7.5%	0.029	0.974	0.024	0.024	0.002	97.4
	(2.75)	(125.45)	(0.57)	(3.44)	(3.16)	
7.5% to 10%	0.006	0.976	0.149	0.018	0.001	97.5
	(0.36)	(64.67)	(2.39)	(1.88)	(1.54)	
Panel B: 31-60 days						
-10% to -7.5%	0.131	0.964	-0.503	0.062	0.008	95.1
	(1.42)	(183.29)	(-3.01)	(0.76)	(0.90)	
-7.5% to -5%	0.243	0.969	-0.475	0.137	0.015	95.7
	(4.37)	(224.71)	(-4.06)	(2.76)	(2.70)	
-5% to -2.5%	0.253	0.964	-0.613	0.107	0.009	95.8
	(7.11)	(234.88)	(-6.23)	(3.40)	(2.97)	
-2.5% to 0%	0.103	0.969	-0.364	0.051	0.006	96.7
	(3.86)	(225.42)	(-3.87)	(2.64)	(2.83)	
0% to 2.5%	0.095	0.972	-0.228	0.060	0.006	97.0
	(5.51)	(235.57)	(-4.99)	(4.08)	(3.92)	
2.5% to 5%	0.074	0.965	-0.171	0.049	0.005	96.9
	(5.88)	(160.65)	(-4.28)	(4.66)	(4.49)	
5% to 7.5%	0.034	0.955	-0.057	0.033	0.003	95.9
	(2.70)	(128.50)	(-1.28)	(3.39)	(3.46)	
7.5% to 10%	0.015	0.945	0.030	0.022	0.002	95.3
	(1.12)	(79.96)	(0.54)	(2.12)	(2.20)	

Table 9. Pre- and post-2008 results for delta-hedged gains: S&P 500 index call options

This table repeats the analysis in Table 6 for two sub-periods: the first spans 1996 to 2008, and the second covers 2009 to 2014. The table reports the estimation results for call options from the regression of delta-hedged gains on the lagged delta-hedged gains, the risk-neutral volatility, skewness, and kurtosis:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \beta_3 SKEW_t + \beta_4 KURT_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t, $SKEW_t$, and $KURT_t$ are the model-free estimates of the risk-neutral volatility, skewness, and kurtosis, respectively, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The estimates for the intercept and the coefficient on the lagged gains are not reported for simplicity. The regressions are performed separately for each maturity and moneyness category. The moneyness of the option is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with maturity 14-30 days and Panel B for options with maturity 31-60 days.

Moneyness y	Pre-2008			Post-2008		
	VOL	SKEW	KURT	VOL	SKEW	KURT
Panel A: 14-30 days						
-10% to -7.5%	-0.565	-0.063	-0.005	-0.171	0.056	0.005
	(-3.37)	(-1.98)	(-1.02)	(-1.79)	(2.20)	(2.48)
-7.5% to -5%	-0.221	-0.054	-0.005	-0.109	0.029	0.002
	(-2.52)	(-1.88)	(-1.24)	(-1.91)	(2.40)	(2.44)
-5% to -2.5%	-0.069	-0.079	-0.008	-0.141	0.035	0.003
	(-0.72)	(-2.63)	(-2.09)	(-1.94)	(2.43)	(2.46)
-2.5% to 0%	-0.052	-0.131	-0.015	-0.178	0.035	0.003
	(-0.52)	(-3.33)	(-2.76)	(-2.31)	(1.84)	(1.73)
0% to 2.5%	-0.028	-0.193	-0.025	-0.179	0.009	0.000
	(-0.33)	(-3.77)	(-3.01)	(-2.37)	(0.43)	(0.12)
2.5% to 5%	-0.043	-0.214	-0.029	-0.193	-0.011	-0.002
	(-0.40)	(-3.57)	(-2.87)	(-2.66)	(-0.53)	(-1.12)
5% to 7.5%	-0.070	-0.223	-0.035	-0.068	-0.117	-0.010
	(-0.46)	(-3.92)	(-3.77)	(-0.72)	(-2.32)	(-2.68)
7.5% to 10%	-0.012	-0.029	-0.007	0.137	-0.267	-0.024
	(-0.10)	(-0.29)	(-0.52)	(0.58)	(-1.84)	(-2.06)
Panel B: 31-60 days						
-10% to -7.5%	-0.697	-0.086	-0.012	-0.356	-0.002	0.000
	(-8.63)	(-3.70)	(-2.87)	(-3.68)	(-0.12)	(-0.05)
-7.5% to -5%	-0.408	-0.066	-0.008	-0.237	-0.014	-0.001
	(-6.94)	(-4.68)	(-4.11)	(-3.10)	(-0.88)	(-0.76)
-5% to -2.5%	-0.303	-0.077	-0.010	-0.215	-0.031	-0.002
	(-5.22)	(-4.72)	(-4.05)	(-2.63)	(-1.37)	(-1.36)
-2.5% to 0%	-0.230	-0.109	-0.014	-0.201	-0.010	-0.001
	(-4.78)	(-4.43)	(-3.71)	(-2.10)	(-0.41)	(-0.52)
0% to 2.5%	-0.166	-0.174	-0.024	-0.309	0.001	0.000
	(-2.84)	(-4.97)	(-4.02)	(-3.30)	(0.03)	(0.04)
2.5% to 5%	-0.150	-0.203	-0.027	-0.090	0.018	0.001
	(-2.21)	(-5.16)	(-4.16)	(-0.91)	(0.63)	(0.57)
5% to 7.5%	-0.146	-0.220	-0.030	-0.161	0.021	0.000
	(-1.91)	(-4.83)	(-4.08)	(-1.48)	(0.63)	(0.09)
7.5% to 10%	-0.076	-0.193	-0.028	-0.234	-0.045	-0.007
	(-0.77)	(-4.04)	(-3.97)	(-0.97)	(-0.82)	(-1.46)

Table 10. Pre- and post-2008 results for delta-hedged gains: S&P 500 index put options

This table repeats the analysis in Table 7 for two sub-periods: the first spans 1996 to 2008, and the second covers 2009 to 2014. The table reports the estimation results for put options from the regression of delta-hedged gains on the lagged delta-hedged gains, the risk-neutral volatility, skewness, and kurtosis:

$$GAINS_t = \beta_0 + \beta_1 GAINS_{t-1} + \beta_2 VOL_t + \beta_3 SKEW_t + \beta_4 KURT_t + \varepsilon_t,$$

where $GAINS_t \equiv \pi_{t,t+\tau}/S_t$ and VOL_t, $SKEW_t$, and $KURT_t$ are the model-free estimates of the risk-neutral volatility, skewness, and kurtosis, respectively, constructed following Bakshi, Kapadia, and Madan (2003). The reported values are the coefficient estimates, the corresponding t-statistics (in parenthesis), and the adjusted R^2. The t-statistics are based on the Newey-West procedures with a lag length of 30. The estimates for the intercept and coefficient on the lagged gains are not reported for simplicity. The regressions are performed separately for each maturity and moneyness group. The moneyness is defined as $y \equiv Se^{(r-\delta)\tau}/K - 1$. Panel A is for options with maturity 14-30 days and Panel B for options with maturity 31-60 days.

Moneyness y	Pre-2008			Post-2008		
	VOL	SKEW	KURT	VOL	SKEW	KURT
Panel A: 14-30 days						
-10% to -7.5%	-0.775	0.660	0.106	-0.117	0.190	0.021
	(-4.17)	(1.80)	(1.73)	(-0.43)	(1.56)	(1.98)
-7.5% to -5%	-0.642	0.392	0.069	-0.432	0.152	0.015
	(-3.54)	(2.08)	(1.89)	(-4.20)	(2.71)	(2.80)
-5% to -2.5%	-0.596	0.201	0.058	-0.301	0.084	0.010
	(-3.57)	(1.41)	(1.99)	(-3.04)	(1.92)	(2.31)
-2.5% to 0%	-0.114	0.147	0.026	-0.123	0.080	0.006
	(-1.04)	(2.39)	(2.49)	(-1.85)	(4.30)	(4.18)
0% to 2.5%	0.010	0.115	0.018	-0.123	0.076	0.006
	(0.15)	(3.29)	(2.89)	(-2.10)	(4.23)	(4.06)
2.5% to 5%	0.019	0.075	0.011	-0.121	0.058	0.004
	(0.29)	(3.51)	(2.86)	(-2.47)	(3.67)	(3.34)
5% to 7.5%	0.035	0.043	0.006	0.015	0.036	0.003
	(0.59)	(2.96)	(2.55)	(0.36)	(3.43)	(3.02)
7.5% to 10%	0.167	0.035	0.004	0.080	0.020	0.001
	(2.08)	(1.91)	(1.64)	(0.94)	(1.75)	(1.37)
Panel B: 31-60 days						
-10% to -7.5%	-0.584	0.269	0.065	-0.323	0.122	0.014
	(-2.52)	(1.01)	(1.45)	(-1.30)	(1.29)	(1.48)
-7.5% to -5%	-0.497	0.363	0.071	-0.421	0.187	0.018
	(-2.92)	(3.13)	(3.56)	(-2.24)	(3.46)	(3.08)
-5% to -2.5%	-0.692	0.244	0.043	-0.282	0.098	0.009
	(-5.23)	(3.58)	(4.05)	(-2.69)	(3.07)	(3.03)
-2.5% to 0%	-0.378	0.144	0.023	-0.312	0.061	0.005
	(-2.99)	(3.96)	(3.82)	(-3.20)	(2.18)	(1.79)
0% to 2.5%	-0.207	0.121	0.018	-0.274	0.079	0.006
	(-3.46)	(4.57)	(4.13)	(-3.38)	(3.51)	(2.96)
2.5% to 5%	-0.147	0.088	0.013	-0.245	0.069	0.005
	(-2.93)	(4.82)	(4.17)	(-3.08)	(2.96)	(2.34)
5% to 7.5%	-0.045	0.068	0.010	-0.081	0.042	0.003
	(-0.90)	(4.28)	(3.90)	(-0.90)	(1.76)	(1.42)
7.5% to 10%	-0.005	0.045	0.006	0.060	0.022	0.002
	(-0.11)	(3.00)	(2.68)	(0.55)	(0.98)	(0.74)

Figure 1. Daily option implied moments of S&P 500 index returns

The figure plots the time series of daily option implied volatility, skewness, and kurtosis for the S&P 500 index return from January 1996 through December 2014. To estimate option implied moments, we apply the model-free methodology proposed in Bakshi, Kapadia, and Madan (2003) to the S&P 500 index options data available from OptionMetrics Database. Details of the methodology and implementation are documented in Section 2.2.